Topic- 11
The Analysis of Variance
Experimental Design

• The sampling plan or experimental design determines the way that a sample is selected.
• In an observational study, the experimenter observes data that already exist. The sampling plan is a plan for collecting this data.
• In a designed experiment, the experimenter imposes one or more experimental conditions on the experimental units and records the response.
Experimental Design

- Must design an experiment that will test your hypothesis.
- This experiment will allow you to change some conditions or variables to test your hypothesis.

Figure 1-1  General model of a process or system.
Variables

Variables are things that change.

The *independent variable* is the variable that is purposely changed. It is the manipulated variable.

The *dependent variable* changes in response to the independent variable. It is the responding variable.
The Basic Principles of Experimental Design

- Two aspects to any experimental problem:
  1. The design of the experiment
  2. Statistical analysis of the data
- Three basic principles of experimental design
  1. Replication
  2. Randomization
  3. Blocking

To reduce error
Definitions

- An **experimental unit** is the object on which a measurement or measurements) is taken.
- A **factor** is an independent variable whose values are controlled and varied by the experimenter.
- A **level** is the intensity setting of a factor.
- A **treatment** is a specific combination of factor levels.
- The **response** is the variable being measured by the experimenter.
Example

- A group of people is randomly divided into an experimental and a control group. The control group is given an aptitude test after having eaten a full breakfast. The experimental group is given the same test without having eaten any breakfast.

<table>
<thead>
<tr>
<th>Experimental unit</th>
<th>Response</th>
<th>Treatments</th>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>person</td>
<td>Score on test</td>
<td>Breakfast or no breakfast</td>
<td>meal</td>
<td>Breakfast or no breakfast</td>
</tr>
</tbody>
</table>
The experimenter in the previous example also records the person’s gender. Describe the factors, levels and treatments.

**Experimental unit** = person

**Response** = score

**Factor #1** = meal

**Factor #2** = gender

**Levels** = breakfast or no breakfast

**Levels** = male or female

**Treatments:**
- male and breakfast, female and breakfast, male and no breakfast, female and no breakfast
The Analysis of Variance (ANOVA)

- All measurements exhibit **variability**.
- The total variation in the response measurements is broken into portions that can be attributed to various **factors**.
- These portions are used to judge the effect of the various factors on the experimental response.

*Figure 1-1*   General model of a process or system.
The Analysis of Variance

- If an experiment has been properly designed,

![Diagram showing total variation and random variation]

The variation between the sample means is larger than the typical variation within the samples.

The variation between the sample means is about the same as the typical variation within the samples.
Assumptions

1. The observations within each population are normally distributed with a common variance $\sigma^2$.
2. Assumptions regarding the sampling procedures are specified for each design.

Analysis of variance procedures are fairly robust when sample sizes are equal and when the data are fairly mound-shaped.
Two Designs

1. **Completely Randomized Design (CRD)**
   an extension of the two independent sample $t$-test.

2. **Completely Randomized Block Design (CRBD)**
   an extension of the paired difference test.
1. The Completely Randomized Design (CRD)

- A one-way classification in which one factor is set at \( k \) different levels.
- The \( k \) levels correspond to \( k \) different normal populations, which are the treatments.
- Are the \( k \) population means the same, or is at least one mean different from the others?
Randomization in CRD

- Factor: A
- A has 4 levels (a₁, a₂, a₃, and a₄)
- There are 4 replications each level (balance)
- So, there will be 4 x 4 = 16 EU
Assumptions

1. Randomness & Independence of Errors
   - Independent Random Samples are Drawn for each condition

2. Normality
   - Populations (for each condition) are Normally Distributed

3. Homogeneity of Variance
   - Populations (for each condition) have Equal Variances
About CRD

- Random samples of size $n_1, n_2, ..., n_k$ are drawn from $k$ populations with means $\mu_1, \mu_2, ..., \mu_k$ and with common variance $\sigma^2$.
- Let $x_{ij}$ be the $j$-th measurement (replication) and the $i$-th sample.
- The total variation in the experiment is measured by the **total sum of squares**:

$$\text{Total SS} = \sum (x_{ij} - \bar{x})^2$$
Example

Is the attention duration of children affected by whether or not they had a good breakfast? Twelve children were randomly divided into three groups and assigned to a different meal plan. The response was attention duration in minutes during the morning reading time.

<table>
<thead>
<tr>
<th>No Breakfast</th>
<th>Light Breakfast</th>
<th>Full Breakfast</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 (X_{11})</td>
<td>14 (X_{21})</td>
<td>10 (X_{31})</td>
</tr>
<tr>
<td>7 (X_{12})</td>
<td>16 (X_{22})</td>
<td>12 (X_{32})</td>
</tr>
<tr>
<td>9 (X_{13})</td>
<td>12 (X_{23})</td>
<td>16 (X_{33})</td>
</tr>
<tr>
<td>13 (X_{14})</td>
<td>17 (X_{24})</td>
<td>15 (X_{34})</td>
</tr>
</tbody>
</table>

$k = 3$ treatments. Are the average attention spans different?
The ANOVA Table of the CRD

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>$k - 1$</td>
<td>SST</td>
<td>SST/($k-1$)</td>
<td>MST/MSE</td>
</tr>
<tr>
<td>Error</td>
<td>$n - k$</td>
<td>SSE</td>
<td>SSE/($n-k$)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$n - 1$</td>
<td>Total SS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### SS(Sum of Squares)

\[
SST = \sum_{i=1}^{k} \frac{T_i^2}{n_i} - \left( \frac{\sum x_{ij}}{n} \right)^2
\]

Total SS = \[ \sum x_{ij}^2 - \left( \frac{\sum x_{ij}}{n} \right)^2 \]

### MS(Mean Squares)

\[
MST = \frac{SST}{(k-1)}
\]

\[
MSE = \frac{SSE}{(n-k)}
\]

\[
SSE = \text{Total SS} - \text{SST}
\]
Computing Formulas

Total SS = \sum x_{ij}^2 - CM
     = (Sum of squares of all x-values) - CM

with

CM = \frac{(\sum x_{ij})^2}{n} = \frac{G^2}{n}
SST = \sum \frac{T_i^2}{n_i} - CM
SSE = Total SS - SST

and

G = Grand total of all n observations
T_i = Total of all observations in sample i
n_i = Number of observations in sample i
n = n_1 + n_2 + \cdots + n_k

Note:
CM = Common Mean
CM = CF
CF = Correction Factor
The Total SS is divided into two parts:

- **SST** (sum of squares for treatments): measures the variation among the $k$ sample means.
- **SSE** (sum of squares for error): measures the variation within the $k$ samples.

$$\text{Total SS} = \text{SST} + \text{SSE}$$

- These **sums of squares** behave like the numerator of a sample variance. When divided by the appropriate **degrees of freedom**, each provides a **mean square**, an estimate of variation in the experiment.
- **Degrees of freedom** are additive, just like the sums of squares.

$$\text{Total } df = \text{Trt } df + \text{Error } df$$
The Breakfast Problem

<table>
<thead>
<tr>
<th>No Breakfast</th>
<th>Light Breakfast</th>
<th>Full Breakfast</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>13</td>
<td>17</td>
<td>15</td>
</tr>
</tbody>
</table>

\[ T_1 = 37 \]
\[ T_2 = 59 \]
\[ T_3 = 53 \]

\[ G = 149 \]

\[ CM = \frac{149^2}{12} = 1850.0833 \]

\[ \text{Total } SS = 8^2 + 7^2 + \ldots + 15^2 - CM = 1973 - 1850.0833 = 122.9167 \]

\[ \text{SST} = \frac{37^2}{4} + \frac{59^2}{4} + \frac{53^2}{4} - CM = 1914.75 - CM = 64.6667 \]

\[ \text{SSE} = \text{Total SS} - \text{SST} = 58.25 \]
The Breakfast Problem

\[ CM = \frac{149^2}{12} = 1850.0833 \]

Total SS = \(8^2 + 7^2 + \ldots + 15^2 - CM = 1973 - 1850.0833 = 122.9167\]

\[ SST = \frac{37^2}{4} + \frac{53^2}{4} + \frac{59^2}{4} - CM = 1914.75 - CM = 64.6667 \]

\[ SSE = \text{Total SS} - \text{SST} = 58.25 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>2</td>
<td>64.6667</td>
<td>32.3333</td>
<td>5.00</td>
</tr>
<tr>
<td>Error</td>
<td>9</td>
<td>58.25</td>
<td>6.4722</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>122.9167</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Testing the Treatment Means

\[ H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k \] versus
\[ H_1 : \text{at least one mean is different} \]

Remember that \( \sigma^2 \) is the common variance for all \( k \) populations. The quantity \( \text{MSE} = \frac{\text{SSE}}{(n - k)} \) is a pooled estimate of \( \sigma^2 \), a weighted average of all \( k \) sample variances, whether or not \( H_0 \) is true.
If $H_0$ is true, then the variation in the sample means, measured by MST = $[\text{SST}/ (k - 1)]$, also provides an unbiased estimate of $\sigma^2$.

However, if $H_0$ is false and the population means are different, then MST— which measures the variance in the sample means — is unusually large. The test statistic $F = \text{MST}/ \text{MSE}$ tends to be larger than usual.
The F Test

- Hence, you can reject $H_0$ for large values of $F$, using a right-tailed statistical test.
- When $H_0$ is true, this test statistic has an $F$ distribution with $df_1 = (k - 1)$ and $df_2 = (n - k)$ degrees of freedom and right-tailed critical values of the $F$ distribution can be used.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$

Test Statistic $: F = \frac{\text{MST}}{\text{MSE}}$

Reject $H_0$ if $F > F_\alpha$ with $k - 1$ and $n-k$ df.
The Breakfast Problem

We reject $H_0$ and conclude that there is a difference in average of attention spans.
Confidence Intervals

If a difference exists between the treatment means, we can explore it with confidence intervals.

A single mean, \( \mu_i : \bar{x}_i \pm t_{\alpha/2} \frac{s}{\sqrt{n_i}} \)

Difference \( \mu_i - \mu_j : (\bar{x}_i - \bar{x}_j) \pm t_{\alpha/2} \sqrt{s^2 \left( \frac{1}{n_i} + \frac{1}{n_j} \right)} \)

where \( s = \sqrt{\text{MSE}} \) and \( t \) is based on error \( df \).
Tukey’s Method for Paired Comparisons

• Designed to test all pairs of population means simultaneously, with an overall error rate of $\alpha$.
• Based on the studentized range, the difference between the largest and smallest of the $k$ sample means.
• Assume that the sample sizes are equal and calculate a “ruler” that measures the distance required between any pair of means to declare a significant difference.
Tukey’s Method

Calculate: \[ \omega = q_\alpha (k, df) \frac{s}{\sqrt{n_i}} \]

where \( k = \text{number of treatment means} \)
\( s = \sqrt{\text{MSE}} \quad df = \text{error df} \)
\( n_i = \text{common sample size} \)
\( q_\alpha (k, n - k) = \text{value from Table 11} \).

If any pair of means differ by more than \( \omega \), they are declared different.
The Breakfast Problem

Use Tukey’s method to determine which of the three population means differ from the others.

<table>
<thead>
<tr>
<th></th>
<th>No Breakfast</th>
<th>Light Breakfast</th>
<th>Full Breakfast</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 ) = 37</td>
<td></td>
<td>59</td>
<td>53</td>
</tr>
<tr>
<td>Means ( \frac{37}{4} = 9.25 )</td>
<td>( \frac{59}{4} = 14.75 )</td>
<td>( \frac{53}{4} = 13.25 )</td>
<td></td>
</tr>
</tbody>
</table>

\[
\omega = q_{0.05} (3,9) \frac{s}{\sqrt{4}} = 3.95 \frac{\sqrt{6.4722}}{\sqrt{4}} = 5.02
\]
The Breakfast Problem

List the sample means from smallest to largest.

\[
\bar{x}_1 \quad \bar{x}_3 \quad \bar{x}_2
\]

\[
9.25 \quad 13.25 \quad 14.75
\]

Since the difference between 9.25 and 13.25 is less than \( \omega = 5.02 \), there is no significant difference. There is a difference between population means 1 and 2 however…

There is no difference between 13.25 and 14.75.

We can declare a significant difference in average attention spans between “no breakfast” and “light breakfast”, but not between the other pairs.
2. THE COMPLETELY RANDOMIZED BLOCK DESIGN (CRBD)

- A direct extension of the paired difference or matched pairs design.

- A **two-way classification** in which \( k \) treatment means are compared.

- The design uses **blocks** of \( k \) experimental units that are relatively similar or homogeneous, with one unit within each block randomly assigned to each treatment.
**RANDOMIZATION IN CRBD**

- **Factor**: A
- **A** has 4 levels (a₁, a₂, a₃, and a₄)
- **Block**: 4 blocks *(in CRD, this is like replication)*
- So, there will be 4 x 4 = 16 EU

![Diagram showing randomization in CRBD with 4 blocks and 4 levels for factor A]
Let $x_{ij}$ be the response for the $i$-th treatment applied to the $j$-th block.

$i = 1, 2, \ldots, k$ \quad $j = 1, 2, \ldots, b$

The total variation in the experiment is measured by the **total sum of squares**:

$$\text{Total SS} = \sum(x_{ij} - \bar{x})^2$$
# The ANOVA Table of the CRBD

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>$k-1$</td>
<td>SST</td>
<td>SST/$(k-1)$</td>
<td>MST/MSE</td>
</tr>
<tr>
<td>Blocks</td>
<td>$b-1$</td>
<td>SSB</td>
<td>SSB/$(b-1)$</td>
<td>MSB/MSE</td>
</tr>
<tr>
<td>Error</td>
<td>$(b-1)(k-1)$</td>
<td>SSE</td>
<td>SSE/$(b-1)(k-1)$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$n-1$</td>
<td>Total SS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SS (Sum of Squares)**

$$SST = \frac{\sum_{i=1}^{k} T_i^2}{n} - \frac{(\sum x_{ij})^2}{b}$$

$$SSB = \frac{\sum_{j=1}^{b} B_j^2}{k} - \frac{(\sum x_{ij})^2}{n}$$

Total SS = $\sum x_{ij}^2 - \frac{(\sum x_{ij})^2}{n}$

**SSE = Total SS - SST - SSB**

**MS (Mean Squares)**

$$MST = \frac{SST}{(k-1)}$$

$$MSB = \frac{SSB}{(b-1)}$$

$$MSE = \frac{SSE}{(n-k)}$$
The **Total SS** is divided into 3 parts:

- **SST** (sum of squares for treatments): measures the variation among the \( k \) treatment means
- **SSB** (sum of squares for blocks): measures the variation among the \( b \) block means
- **SSE** (sum of squares for error): measures the random variation or experimental error

in such a way that:

\[
\text{Total SS} = \text{SST} + \text{SSB} + \text{SSE}
\]

\[
df \text{ Total} = df T + df B + df E
\]
COMPUTING FORMULAS

\[
CM = \frac{G^2}{n} \quad \text{where} \quad G = \sum x_{ij}
\]

Total SS = \( \sum x_{ij}^2 - CM \)

\[
SST = \frac{\sum T_i^2}{b} - CM \quad \text{where} \quad T_i = \text{total for treatment } i
\]

\[
SSB = \frac{\sum B_j^2}{k} - CM \quad \text{where} \quad B_j = \text{total for block } j
\]

SSE = Total SS - SST - SSB
THE SEEDLING PROBLEM

### Locations

<table>
<thead>
<tr>
<th>Soil Prep</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11</td>
<td>13</td>
<td>16</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>12</td>
<td>64</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>15</td>
<td>13</td>
<td>10</td>
<td>48</td>
</tr>
<tr>
<td>$B_j$</td>
<td>36</td>
<td>45</td>
<td>49</td>
<td>32</td>
<td>162</td>
</tr>
</tbody>
</table>

\[
\text{CM} = \frac{162^2}{12} = 2187
\]

\[
\text{Total SS} = \left(11^2 + 15^2 + \ldots + 10^2\right) - 2187 = 111
\]

\[
\text{SST} = \frac{50^2 + 64^2 + 48^2}{4} - 2187 = 38
\]

\[
\text{SSB} = \frac{36^2 + 45^2 + 49^2 + 32^2}{3} - 2187 = 61.6667
\]

\[
\text{SSE} = 111 - 38 - 61.6667 = 11.3333
\]
THE SEEDLING PROBLEM - ANOVA

Total df = $bk - 1 = n - 1$

Treatment df = $k - 1$

Block df = $b - 1$

Error df = $bk - (k - 1) - (b - 1) = (k-1)(b-1)$

Mean Squares

- $MST = \frac{SST}{k-1}$
- $MSB = \frac{SSB}{b-1}$
- $MSE = \frac{SSE}{(k-1)(b-1)}$

Source | df | SS | MS | F
---|---|---|---|---
Treatments | $k - 1$ | SST | $\frac{SST}{(k-1)}$ | MST/MSE
Blocks | $b - 1$ | SSB | $\frac{SSB}{(b-1)}$ | MSB/MSE
Error | $(b-1)(k-1)$ | SSE | $\frac{SSE}{(b-1)(k-1)}$ | 
Total | $n - 1$ | Total SS | 

THE SEEDLING PROBLEM-ANOVA
THE SEEDLING PROBLEM-ANOVA

\[
CM = \frac{162^2}{12} = 2187
\]

Total SS = \(11^2 + 15^2 + \ldots + 10^2 - 2187 = 111\)

\[
SST = \frac{50^2 + 64^2 + 48^2}{4} - 2187 = 38
\]

\[
SSB = \frac{36^2 + 45^2 + 49^2 + 32^2}{3} - 2187 = 61.6667
\]

\[
SSE = 111 - 38 - 61.6667 = 11.3333
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>2</td>
<td>38</td>
<td>19</td>
<td>10.06</td>
</tr>
<tr>
<td>Blocks</td>
<td>3</td>
<td>61.667</td>
<td>20.5556</td>
<td>10.88</td>
</tr>
<tr>
<td>Error</td>
<td>6</td>
<td>11.3333</td>
<td>1.8889</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>122.9167</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For either treatment or block means, we can test:

\[ H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots \quad \text{versus} \]
\[ H_1 : \text{at least one mean is different} \]

To test the \( H_0 \) that treatment (or block) means are equal:

Test Statistic: \( F = \frac{\text{MST}}{\text{MSE}} \) (or \( F = \frac{\text{MSB}}{\text{MSE}} \))

Reject \( H_0 \) if \( F > F_\alpha \) with \( k - 1 \) (or \( b - 1 \)) and \( (b - 1)(k - 1) \) df.
TESTING THE TREATMENT AND BLOCK MEANS

- Remember that $\sigma^2$ is the common variance for all $bk$ treatment/block combinations. MSE is the best estimate of $\sigma^2$, whether or not $H_0$ is true.

- If $H_0$ is false and the population means are different, then MST or MSB— whichever you are testing— will unusually large.
  
  The test statistic $F = \frac{\text{MST}}{\text{MSE}}$ (or $F = \frac{\text{MSB}}{\text{MSE}}$) tends to be larger than usual.

- We use a right-tailed $F$ test with the appropriate degrees of freedom.
### THE SEEDLING PROBLEM-ANOVA

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil Prep (Trts)</td>
<td>2</td>
<td>38</td>
<td>19</td>
<td>10.06</td>
</tr>
<tr>
<td>Location (Blocks)</td>
<td>3</td>
<td>61.6667</td>
<td>20.5556</td>
<td>10.88</td>
</tr>
<tr>
<td>Error</td>
<td>6</td>
<td>11.3333</td>
<td>1.8889</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>11</td>
<td>122.9167</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To test for a difference due to soil preparation:

- $H_0: \mu_1 = \mu_2 = \mu_3$ versus
- $H_a: \text{at least one mean is different}$

$$F = \frac{\text{MST}}{\text{MSE}} = 10.06$$

Rejection region: $F > F_{0.05} = 5.14$.

We reject $H_0$ and conclude that there is a difference of seedling growth due to soil preparation.

Although not of primary importance, notice that the blocks (locations) were also significantly different ($F = 10.88$).
CONFIDENCE INTERVALS

If a difference exists between the treatment means or block means, we can explore it with confidence intervals or using Tukey’s method.

Difference in treatment means: \[(\overline{T}_i - \overline{T}_j) \pm t_{\alpha/2} \sqrt{s^2 \left(\frac{2}{b}\right)}\]

Difference in block means: \[(\overline{B}_i - \overline{B}_j) \pm t_{\alpha/2} \sqrt{s^2 \left(\frac{2}{k}\right)}\]

where \(\overline{T}_i = T_i / b\) and \(\overline{B}_i = B_i / k\) are the necessary treatment or block means.

\(s = \sqrt{\text{MSE}}\) and \(t\) is based on error \(df\).
**TUKEY’S METHOD**

For comparing treatment means:

\[ \omega = q_\alpha (k, df) \frac{s}{\sqrt{b}} \]

For comparing block means:

\[ \omega = q_\alpha (b, df) \frac{s}{\sqrt{k}} \]

where:

\[ s = \sqrt{\text{MSE}} \quad df = \text{error} \ df \]

\[ q_\alpha (k, df) = \text{value from Table 11.} \]

If any pair of means differ by more than \( \omega \), they are declared different.
Use Tukey’s method to determine which of the three soil preparations differ from the others.

<table>
<thead>
<tr>
<th></th>
<th>A (no prep)</th>
<th>B (fertilization)</th>
<th>C (burning)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>50</td>
<td>64</td>
<td>48</td>
</tr>
<tr>
<td>Means</td>
<td>50/4 = 12.5</td>
<td>64/4 = 16</td>
<td>48/4 = 12</td>
</tr>
</tbody>
</table>

\[
\alpha = q_{0.05}(3, 6) \frac{s}{\sqrt{4}}
\]

\[
= 4.34 \frac{\sqrt{1.8889}}{\sqrt{4}} = 2.98
\]

![Table 11(a)](image)
List the sample means from smallest to largest.

<table>
<thead>
<tr>
<th>$T_C$</th>
<th>$T_A$</th>
<th>$T_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>12.5</td>
<td>16.0</td>
</tr>
</tbody>
</table>

Since the difference between 12 and 12.5 is less than $\omega = 2.98$, there is no significant difference. There is a difference between population means C and B however.

**There is a significant difference between A and B.**

A significant difference in average growth only occurs when the soil has been fertilized.
A randomized block design should not be used when both treatments and blocks correspond to experimental factors of interest to the researcher.

Remember that blocking may not always be beneficial.

Remember that you cannot construct confidence intervals for individual treatment means unless it is reasonable to assume that the $b$ blocks have been randomly selected from a population of blocks.
Normal Probability Plot

- If the normality assumption is valid, the plot should resemble a straight line, sloping upward to the right.
- If not, you will often see the pattern fail in the tails of the graph.
If the equal variance assumption is valid, the plot should appear as a random scatter around the zero center line. If not, you will see a pattern in the residuals.