DEMONSTRATION 4.1

COMPUTING MEASURES OF VARIABILITY

For the following sample data, compute the variance and standard deviation. The scores are:

10,  7,  6,  10,  6,  15

**STEP 1** Compute \( SS \), the sum of squared deviations

We will use the definitional formula: \( SS = \sum (X - M)^2 \). For this sample, the mean is 
\[ M = \frac{\sum X}{n} = \frac{54}{6} = 9 \]. The following table shows the deviation and the squared deviation for each score.

<table>
<thead>
<tr>
<th>X</th>
<th>X - M</th>
<th>(X - M)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>15</td>
<td>+6</td>
<td>36</td>
</tr>
</tbody>
</table>

\( SS \) is the sum of the squared deviations:

\[
SS = \sum (X - M)^2 = 1 + 4 + 9 + 1 + 9 + 36 = 60
\]

**STEP 2** Compute the sample variance

For sample variance, \( SS \) is divided by the degrees of freedom, \( df = n - 1 \).

\[
s^2 = \frac{SS}{n - 1} = \frac{60}{5} = 12
\]

**STEP 3** Compute the sample standard deviation

Standard deviation is simply the square root of the variance.

\[
s = \sqrt{12} = 3.46
\]

PROBLEMS

1. Can \( SS \) ever have a value less than zero? Explain your answer.
2. What does it mean for a sample to have a standard deviation of zero? Describe the scores in such a sample.
3. What does it mean for a sample to have a standard deviation of \( s = 5 \)? Describe the scores in such a sample. (Describe where the scores are located relative to the sample mean.)
4. A population has \( \mu = 100 \) and \( \sigma = 20 \). If you select a single score from this population, on the average, how close would it be to the population mean? Explain your answer.
5. In words, explain what is measured by each of the following:
   a. \( SS \)
   b. Variance
   c. Standard deviation
7. On an exam with \( X = 1 \), you have a score of \( s = 2 \). Is this good or bad?
   a. With respect to the mean, you have scored above the mean.
   b. If your standard deviation is small, you may have \( \sigma \)
   c. Depends on the rank of your \( \sigma \)
8. Explain the difference between range, variance, and standard deviation.
9. For the following:
   3, 4, 7, 8, 12
   a. Can \( SS \) ever have a value less than zero? Explain your answer.
   b. Are you likely to get a score within one standard deviation of the mean?
   c. Are you likely to get a score within two standard deviations of the mean?
10. A normal distribution has \( \mu = 100 \) and \( \sigma = 20 \). If you select a single score from this distribution, on the average, how close would it be to the population mean? Explain your answer.
11. For the following:
   a. First find the mean.
   b. Next, find the variance.
   c. Describe how you would find the mean.
12. A sample of scores is given.
   a. If the sample mean is 100, what is the value of the scores? (One possible answer.)
6. For the data in the following sample:
   1, 4, 3, 6, 2, 7, 8, 3, 7, 2, 4, 3
   a. Find the median and the semi-interquartile range.
      (It may help to sketch the frequency distribution histogram.)
   b. Now change the score of \( X = 8 \) to \( X = 18 \), and
      find the new median and semi-interquartile range.
   c. Describe how one extreme score influences the
      median and semi-interquartile range.

7. On an exam with \( \mu = 75 \), you obtain a score of
   \( X = 80 \).
   a. Would you prefer that the exam distribution had
      \( \sigma = 2 \) or \( \sigma = 10 \)? (Sketch each distribution, and
      locate the position of \( X = 80 \) in each one.)
   b. If your score is \( X = 70 \), would you prefer \( \sigma = 2 \) or
      \( \sigma = 10 \)? (Again, sketch each distribution to determine
      how the value of \( \sigma \) affects your position relative to the rest of
      the class.)

8. Explain what it means to say that the sample variance provides an
   unbiased estimate of the population variance.

9. For the following population of \( N = 8 \) scores:
   3, 3, 5, 1, 4, 3, 2, 3
   a. Calculate the range, the interquartile range, and the
      standard deviation.
   b. Add 2 points to every score, then compute the
      range, the interquartile range, and the standard
deviation again. How is variability affected by
      adding a constant to every score?

10. A normal-shaped population has a mean of \( \mu = 80 \)
    and a standard deviation of \( \sigma = 20 \).
    a. If 10 points were added to every score in the
       population, what would be the new values for the
       population mean and standard deviation?
    b. If every score in the original population was multi-
       plied by 2, what would be the new values for the
       population mean and standard deviation?

11. For the data in the following sample:
   8, 1, 5, 1, 5
   a. Find the mean and the standard deviation.
   b. Now change the score of \( X = 8 \) to \( X = 18 \), and
      find the new mean and standard deviation.
   c. Describe how one extreme score influences the
      mean and standard deviation.

12. A sample of \( n = 20 \) scores has a mean of \( M = 30 \).
    a. If the sample standard deviation is \( s = 10 \), would a
       score of \( X = 38 \) be considered an extreme value
       (out in the tail of the distribution)?

b. If the sample standard deviation is \( s = 2 \), would a
   score of \( X = 38 \) be considered an extreme value
   (out in the tail of the distribution)?

13. There are two different formulas or methods that can
    be used to calculate \( SS \).
    a. Under what circumstances is the definitional for-
       mula easy to use?
    b. Under what circumstances is the computational
       formula preferred?

14. For the following population of \( N = 5 \) scores:
   11, 2, 0, 8, 4
   a. Sketch a histogram showing the population distri-
      bution.
   b. Locate the value of the population mean in your
      sketch, and make an estimate of the standard devi-
      ation (as done in Example 4.2).
   c. Compute \( SS \), variance, and standard deviation
      for the population. (How well does your estimate
      compare with the actual value of \( \sigma^2 \)?)

15. For the following set of scores:
   1, 7, 1, 1
   a. Calculate \( \Sigma X \) and \( \Sigma X^2 \).
   b. Use the two sums from part a and the compu-
      tational formula to compute \( SS \) for the scores.

16. For the following scores:
   1, 0, 4, 1, 1, 5
   a. Calculate the mean. (Note that the value of the
      mean does not depend on whether the set of scores
      is considered to be a sample or a population.)
   b. Find the deviation for each score, and check that
      the deviations sum to zero.
   c. Square each deviation, and compute \( SS \). (Again,
      note that the value of \( SS \) is independent of whether
      the set of scores is a sample or a population.)

17. Sketch a normal distribution (see Figure 4.5, page 95)
    with \( \mu = 50 \) and \( \sigma = 20 \).
    a. Locate each of the following scores in your sketch,
       and indicate whether you consider each score to be
       an extreme value (high or low) or a central value:
       65, 55, 40, 47
   b. Make another sketch showing a distribution with
      \( \mu = 50 \), but this time with \( \sigma = 2 \). Now locate each
      of the four scores in the new distribution, and indi-
      cate whether they are extreme or central. (Note:
      The value of the standard deviation can have a
      dramatic effect on the location of a score within a
      distribution.)
18. Suppose that a treatment has the effect of increasing everyone’s score by 6 points. Whether this 6-point treatment effect is viewed as a large effect or a small effect depends on the size of the standard deviation. Consider the following two possibilities:
   a. Sketch a normal distribution with a mean of $\mu = 80$ and a standard deviation of $\sigma = 30$. This represents the original population before treatment. On the same figure, sketch a second normal distribution with a mean of $\mu = 86$ and a standard deviation of $\sigma = 30$. This represents the population after the treatment has added 6 points to each person’s score. Does the 6-point treatment effect look like a big effect? (Is there a big difference between the two distributions?)
   b. Now sketch a normal distribution with a mean of $\mu = 80$ and a standard deviation of $\sigma = 3$. This represents the original population before treatment. On the same figure, sketch a second normal distribution with a mean of $\mu = 86$ and a standard deviation of $\sigma = 3$. This represents the population after the treatment has added 6 points to each person’s score. Does the 6-point treatment effect look like a big effect? (Is there a big difference between the two distributions?)

19. For the following sample of $n = 7$ scores:
   12, 1, 10, 6, 3, 3, 7
   a. Sketch a histogram showing the sample distribution.
   b. Locate the value of the sample mean in your sketch, and make an estimate of the standard deviation (as done in Example 4.2).
   c. Compute $SS$, variance, and standard deviation for the sample. (How well does your estimate compare with the actual value of $\sigma$?)

20. Calculate $SS$, variance, and standard deviation for the following sample of $n = 6$ scores: 11, 0, 8, 2, 4, 5.
   (Note: The definitional formula for $SS$ works well with these scores.)

21. Calculate $SS$, variance, and standard deviation for the following sample of $n = 9$ scores: 2, 0, 0, 0, 0, 2, 0, 2, 0.
   (Note: The computational formula for $SS$ works best with these scores.)

22. Calculate $SS$, variance, and standard deviation for the following sample of $n = 4$ scores: 3, 1, 1, 1.
   (Note: The computational formula works best with these scores.)

23. For the following population of $N = 8$ scores:
   10, 1, 2, 5, 9, 7, 2, 4
   a. Sketch a histogram showing the population distribution.
   b. Locate the value of the population mean in your sketch, and make an estimate of the standard deviation (as done in Example 4.2).
   c. Compute $SS$, variance, and standard deviation for the population. (How well does your estimate compare with the actual value of $\sigma$?)

24. Calculate $SS$, variance, and standard deviation for the following population of $N = 8$ scores: 0, 1, 0, 3, 6, 0, 2, 0. (Note: The computational formula for $SS$ works well with these scores.)

25. Calculate $SS$, variance, and standard deviation for the following sample of $n = 4$ scores: 4, 0, 1, 1. (Note: The computational formula for $SS$ works well with these scores.)

26. For the following population of $N = 6$ scores:
   5, 0, 9, 3, 8, 5
   a. Sketch a histogram showing the population distribution.
   b. Locate the value of the population mean in your sketch, and make an estimate of the standard deviation (as done in Example 4.2).
   c. Compute $SS$, variance, and standard deviation for the population. (How well does your estimate compare with the actual value of $\sigma$?)

27. A recent study reports that older adults who got regular physical exercise (as measured by a pedometer) experienced fewer symptoms of depression, even when tested 2 years later (Fukukawa, Nakashima, Tsubio, Kozakai, Doyo, Naokira, Ando, & Shimokata, 2004). Following are hypothetical data similar to the results obtained in this study.
   a. Calculate the mean and standard deviation for each group of scores.
   b. Based on the statistics from part a, does there appear to be a difference between the two groups?

<table>
<thead>
<tr>
<th>Regular Exercise</th>
<th>No Regular Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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<tr>
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</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>