Tools You Will Need
The following items are considered essential background material for this chapter. If you doubt your knowledge of any of these items, you should review the appropriate chapter or section before proceeding.

- Summation notation (Chapter 1)
- Central tendency (Chapter 3)
  - Mean
  - Median

4.1 Overview
4.2 The Range
4.3 Standard Deviation and Variance for a Population
4.4 Standard Deviation and Variance for Samples
4.5 More about Variance and Standard Deviation

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Focus on Problem Solving
Demonstration 4.1
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4.1 OVERVIEW

The term *variability* has much the same meaning in statistics as it has in everyday language; to say that things are variable means that they are not all the same. In statistics, our goal is to measure the amount of variability for a particular set of scores, a distribution. In simple terms, if the scores in a distribution are all the same, then there is no variability. If there are small differences between scores, then the variability is small, and if there are large differences between scores, then the variability is large.

**Variability** provides a quantitative measure of the differences between scores in a distribution and describes the degree to which the scores are spread out or clustered together.

Figure 4.1 shows two distributions of familiar values: Part (a) shows the distribution of adult male heights (in inches), and part (b) shows the distribution of adult male weights (in pounds). Notice that the two distributions differ in terms of central tendency. The mean height is 70 inches (5 feet, 10 inches) and the mean weight is 170 pounds. In addition, notice that the distributions differ in terms of variability. For example, most heights are clustered close together, within 5 or 6 inches of the mean. On the other hand, weights are spread over a much wider range. In the weight distribution it is not unusual to find individuals who are located more than 30 pounds away from the mean, and it would not be surprising to find two individuals whose weights differ by more than 30 or 40 pounds. The purpose for measuring variability is to obtain an objective measure of how the scores are spread out in a distribution. In general, a good measure of variability serves two purposes:

1. Variability describes the distribution. Specifically, it tells whether the scores are clustered close together or are spread out over a large distance. Usually,

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**FIGURE 4.1**

Population distributions of adult heights and adult weights.

**NOTE:** For simplicity, we have omitted the vertical axis for these graphs. As always, the height of any point on the curve indicates the relative frequency for that particular score.
variability is defined in terms of distance. It tells how much distance to expect between one score and another, or how much distance to expect between an individual score and the mean. For example, we know that most adults’ heights are clustered close together, within 5 or 6 inches of the average. Although more extreme heights exist, they are relatively rare.

2. Variability measures how well an individual score (or group of scores) represents the entire distribution. This aspect of variability is very important for inferential statistics, in which relatively small samples are used to answer questions about populations. For example, suppose that you selected a sample of one person to represent the entire population. Because most adult heights are within a few inches of the population average (the distances are small), there is a very good chance that you would select someone whose height is within 6 inches of the population mean. On the other hand, the scores are much more spread out (greater distances) in the distribution of adult weights. In this case, you probably would not obtain someone whose weight was within 6 pounds of the population mean. Thus, variability provides information about how much error to expect if you are using a sample to represent a population.

In this chapter, we consider three different measures of variability: the range, standard deviation, and the variance. Of these three, the standard deviation and the related measure of variance are by far the most important.

### 4.2 THE RANGE

Continuous and discrete variables were discussed in Chapter 1 on pages 19–21.

The range is the distance covered by the scores in a distribution, from the smallest score to the largest score. When the scores are measurements of a continuous variable, the range can be defined as the difference between the upper real limit (URL) for the largest score ($X_{\text{max}}$) and the lower real limit (LRL) for the smallest score ($X_{\text{min}}$).

$$\text{range} = \text{URL for } X_{\text{max}} - \text{LRL for } X_{\text{min}}$$

If the scores have values from 1 to 5, for example, the range is $5.5 - 0.5 = 5$ points. When the scores are whole numbers, the range is also a measure of the number of measurement categories. If every individual is classified as either 1, 2, 3, 4, or 5, then there are five measurement categories and the range is 5 points.

Defining the range as the number of measurement categories also works for discrete variables that are measured with numerical scores. For example, if you are measuring the number of children in a family and the data produce values from 0 to 4, then there are five measurement categories (0, 1, 2, 3, and 4) and the range is 5 points. By this definition, when the scores are all whole numbers, the range can be obtained by

$$X_{\text{max}} - X_{\text{min}} + 1.$$  

A commonly used alternative definition of the range simply measures the difference between the largest score ($X_{\text{max}}$) and the smallest score ($X_{\text{min}}$), without any reference to real limits.

$$\text{range} = X_{\text{max}} - X_{\text{min}}$$

By this definition, scores having values from 1 to 5 cover a range of only 4 points. Many computer programs, such as SPSS, use this definition. For discrete variables, which do not have real limits, this definition is often considered more appropriate.
Also, this definition works well for variables with precisely defined upper and lower boundaries. For example, if you are measuring proportions of an object, like pieces of a pizza, you can obtain values such as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and so on. Expressed as decimal values, the proportions range from 0 to 1. You can never have a value less than 0 (none of the pizza) and you can never have a value greater than 1 (all of the pizza). Thus, the complete set of proportions is bounded by 0 at one end and by 1 at the other. As a result, the proportions cover a range of 1 point.

Using either definition, the range is probably the most obvious way to describe how spread out the scores are—simply find the distance between the maximum and the minimum scores. The problem with using the range as a measure of variability is that it is completely determined by the two extreme values and ignores the other scores in the distribution. Thus, a distribution with one unusually large (or small) score will have a large range even if the other scores are actually clustered close together.

Because the range does not consider all the scores in the distribution, it often does not give an accurate description of the variability for the entire distribution. For this reason, the range is considered to be a crude and unreliable measure of variability. Therefore, in most situations, it does not matter which definition you use to determine the range.

### 4.3 STANDARD DEVIATION AND VARIANCE FOR A POPULATION

The standard deviation is the most commonly used and the most important measure of variability. Standard deviation uses the mean of the distribution as a reference point and measures variability by considering the distance between each score and the mean.

In simple terms, the standard deviation provides a measure of the standard, or average, distance from the mean, and describes whether the scores are clustered closely around the mean or are widely scattered. The fundamental definition of the standard deviation is the same for both samples and populations, but the calculations differ slightly. We look first at the standard deviation as it is computed for a population, and then turn our attention to samples in Section 4.4.

Although the concept of standard deviation is straightforward, the actual equations appear complex. Therefore, we begin by looking at the logic that leads to these equations. If you remember that our goal is to measure the standard, or typical, distance from the mean, then this logic and the equations that follow should be easier to remember.

**Definition**

The first step in finding the standard distance from the mean is to determine the deviation, or distance from the mean, for each individual score. By definition, the deviation for each score is the difference between the score and the mean.

<table>
<thead>
<tr>
<th>Deviation is distance from the mean:</th>
</tr>
</thead>
<tbody>
<tr>
<td>deviation score $= X - \mu$</td>
</tr>
</tbody>
</table>

For a distribution of scores with $\mu = 50$, if your score is $X = 53$, then your deviation score is

$$X - \mu = 53 - 50 = 3$$
If your score is $X = 45$, then your deviation score is

$$X - \mu = 45 - 50 = -5$$

Notice that there are two parts to a deviation score: the sign (+ or −) and the number. The sign tells the direction from the mean—that is, whether the score is located above (+) or below (−) the mean. The number gives the actual distance from the mean. For example, a deviation score of −6 corresponds to a score that is below the mean by 6 points.

**STEP 2**

Because our goal is to compute a measure of the standard distance from the mean, the obvious next step is to calculate the mean of the deviation scores. To compute this mean, you first add up the deviation scores and then divide by $N$. This process is demonstrated in the following example.

**EXAMPLE 4.1**

We start with the following set of $N = 4$ scores. These scores add up to $\Sigma X = 12$, so the mean is $\mu = \frac{12}{4} = 3$. For each score, we have computed the deviation.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$X - \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>+5</td>
</tr>
<tr>
<td>1</td>
<td>−2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>−3</td>
</tr>
</tbody>
</table>

Note that the deviation scores add up to zero. This should not be surprising if you remember that the mean serves as a balance point for the distribution. The total of the distances above the mean is exactly equal to the total of the distances below the mean (see page 62). Thus, the total for the positive deviations is exactly equal to the total for the negative deviations, and the complete set of deviations always adds up to zero.

Because the sum of the deviations is always zero, the mean of the deviations is also zero and is of no value as a measure of variability. Specifically, the mean of the deviations is zero if the scores are closely clustered and it is zero if the scores are widely scattered. (You should note, however, that the constant value of zero can be useful in other ways. Whenever you are working with deviation scores, you can check your calculations by making sure that the deviation scores add up to zero.)

**STEP 3**

The average of the deviation scores will not work as a measure of variability because it is always zero. Clearly, this problem results from the positive and negative values canceling each other out. The solution is to get rid of the signs (+ and −). The standard procedure for accomplishing this is to square each deviation score. Using the squared values, you then compute the mean squared deviation, which is called variance.

**DEFINITION**

Population variance equals the mean squared deviation. Variance is the average squared distance from the mean.

Note that the process of squaring deviation scores does more than simply get rid of plus and minus signs. It results in a measure of variability based on squared distances. Although variance is valuable for some of the inferential statistical methods covered
later, the concept of squared distance is not an intuitive or easy to understand descriptive measure. For example, it is not particularly useful to know that the squared distance from New York City to Boston is 26,244 miles squared. Therefore, we continue the process one more step.

**STEP 4**
Remember that our goal is to compute a measure of the standard distance from the mean. Variance, which measures the average squared distance from the mean, is not exactly what we want. The final step simply makes a correction for having squared all the distances. The new measure, the standard deviation, is the square root of the variance.

**DEFINITION**

\[
\text{Standard deviation} = \sqrt{\text{variance}}
\]

Figure 4.2 shows the overall process of computing variance and standard deviation. Remember that our goal is to measure variability by finding the standard distance from the mean. However, we cannot simply calculate the average of the distances because this value will always be zero. Therefore, we begin by squaring each distance, then we find the average of the squared distances, and finally we take the square root to obtain a measure of the standard distance. Technically, the standard deviation is the square root of the average squared deviation. Conceptually, however, the standard deviation provides a measure of the average distance from the mean.

Because the standard deviation and variance are defined in terms of distance from the mean, these measures of variability are used only with numerical scores that are obtained from measurements on an interval or a ratio scale. Recall from Chapter 1 (page 23) that these two scales are the only ones that provide information about distance; nominal and ordinal scales do not. Also, recall from Chapter 3 (page 77) that it is inappropriate to compute a mean for ordinal data and it is impossible to compute a mean for nominal data. Because the mean is a critical component in the calculation of standard deviation and
variance, the same restrictions that apply to the mean also apply to these two measures of variability. Specifically, the mean, the standard deviation, and the variance should be used only with numerical scores from interval or ordinal scales of measurement.

Although we still have not presented any formulas for variance or standard deviation, you should be able to compute these two statistical values from their definitions. The following example demonstrates this process.

**Example 4.2**

We will calculate the variance and standard deviation for the following population of \( N = 5 \) scores:

\[
1, \ 9, \ 5, \ 8, \ 7
\]

Remember that the purpose of standard deviation is to measure the standard distance from the mean, so we begin by computing the population mean. These five scores add up to \( \Sigma X = 30 \) so the mean is \( \mu = \frac{30}{5} = 6 \). Next, we find the deviation, (distance from the mean) for each score and then square the deviations. Using the population mean \( \mu = 6 \), these calculations are shown in the following table.

<table>
<thead>
<tr>
<th>Score</th>
<th>Deviation ( X - \mu )</th>
<th>Squared Deviation ( (X - \mu)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ 40 = \text{the sum of the squared deviations} \]

For this set of \( N = 5 \) scores, the squared deviations add up to 40. The mean of the squared deviations, \( \frac{40}{5} = 8 \), and the standard deviation is \( \sqrt{8} = 2.83 \).

You should note that a standard deviation of 2.83 is a sensible answer for this distribution. The five scores in the population are shown in a histogram in Figure 4.3 so that you can see the distances more clearly. Note that the scores closest to the mean are only 1 point away. Also, the score farthest from the mean is 5 points away. For this distribution, the largest distance from the mean is 5 points and the smallest distance is 1 point. Thus, the standard distance should be somewhere between 1 and 5. By looking at a distribution in this way, you should be able to make a rough estimate of the standard deviation. In this case, the standard deviation should be between 1 and 5, probably around 3 points. The value we calculated for the standard deviation is in excellent agreement with this estimate.

Making a quick estimate of the standard deviation can help you avoid errors in calculation. For example, if you calculated the standard deviation for the scores in Figure 4.3 and obtained a value of 12, you should realize immediately that you have made an error. (If the biggest deviation is only 5 points, then it is impossible for the standard deviation to be 12.)
FIGURE 4.3
A frequency distribution histogram for a population of $N = 5$ scores. The mean for this population is $\mu = 6$. The smallest distance from the mean is 1 point, and the largest distance is 5 points. The standard distance (or standard deviation) should be between 1 and 5 points.

FORMULAS FOR POPULATION VARIANCE AND STANDARD DEVIATION

The concepts of standard deviation and variance are the same for both samples and populations. However, the details of the calculations differ slightly, depending on whether you have data from a sample or from a complete population. We first consider the formulas for populations and then look at samples in Section 4.4.

The sum of squared deviations ($SS$) Recall that variance is defined as the mean of the squared deviations. This mean is computed exactly the same way you compute any mean: First find the sum, and then divide by the number of scores.

$$\text{Variance} = \text{mean squared deviation} = \frac{\text{sum of squared deviations}}{\text{number of scores}}$$

The value in the numerator of this equation, the sum of the squared deviations, is a basic component of variability, and we will focus on it. To-simplify things, it is identified by the notation $SS$ (for sum of squared deviations), and it generally is referred to as the sum of squares.

**DEFINITION**

$SS$, or sum of squares, is the sum of the squared deviation scores.

You will need to know two formulas to compute $SS$. These formulas are algebraically equivalent (they always produce the same answer), but they look different and are used in different situations.

The first of these formulas is called the definitional formula because the terms in the formula literally define the process of adding up the squared deviations:

$$\text{Definitional formula: } SS = \sum (X - \mu)^2 \quad (4.1)$$

Following the proper order of operations (page 26), the formula instructs you to perform the following sequence of calculations:

1. Find each deviation score $(X - \mu)$.
2. Square each deviation score, $(X - \mu)^2$.
3. Add the squared deviations.
The result is $SS$, the sum of the squared deviations. The following example demonstrates using this formula.

**Example 4.3**

We will compute $SS$ for the following set of $N = 4$ scores. These scores have a sum of $\Sigma X = 8$, so the mean is $\mu = \frac{8}{4} = 2$. The following table shows the deviation and the squared deviation for each score. The sum of the squared deviation is $SS = 22$.

<table>
<thead>
<tr>
<th>Score</th>
<th>Deviation $X - \mu$</th>
<th>Squared Deviation $(X - \mu)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>−2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>+4</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>−1</td>
<td>$\frac{1}{22} = \Sigma(X - \mu)^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although the definitional formula is the most direct method for computing $SS$, it can be awkward to use. In particular, when the mean is not a whole number, the deviations all contain decimals or fractions, and the calculations become difficult. In addition, calculations with decimal values introduce the opportunity for rounding error, which makes the results less accurate. For these reasons, an alternative formula has been developed for computing $SS$. The alternative, known as the computational formula, performs calculations with the scores (not the deviations) and therefore minimizes the complications of decimals and fractions.

**Computational formula**: $SS = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$  \hspace{1cm} (4.2)

The first part of this formula directs you to square each score and then add the squared values, $\Sigma X^2$. In the second part of the formula, you find the sum of the scores, $\Sigma X$, then square this total and divide the result by $N$. Finally, subtract the second part from the first. The use of this formula is shown in Example 4.4 with the same scores that we used to demonstrate the definitional formula.

**Example 4.4**

The computational formula is used to calculate $SS$ for the same set of $N = 4$ scores we used in Example 4.3. First, compute $\Sigma X$. Then square each score, and compute $\Sigma X^2$. These calculations are shown in the following table. The two sums are used in the formula to compute $SS$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$X^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$SS = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$

$= 38 - \frac{(8)^2}{4}$

$= 38 - \frac{64}{4}$

$= 38 - 16$

$= 22$
Note that the two formulas produce exactly the same value for $SS$. Although the formulas look different, they are in fact equivalent. The definitional formula provides the most direct representation of the concept of $SS$; however, this formula can be awkward to use, especially if the mean includes a fraction or decimal value. If you have a small group of scores and the mean is a whole number, then the definitional formula is fine; otherwise use the computational formula.

With the definition and calculation of $SS$ behind you, the equations for variance and standard deviation become relatively simple. Remember that variance is defined as the mean squared deviation. The mean is the sum divided by $N$, so the equation for the population variance is

$$\text{variance} = \frac{SS}{N}$$

Standard deviation is the square root of variance, so the equation for the population standard deviation is

$$\text{standard deviation} = \sqrt{\frac{SS}{N}}$$

There is one final bit of notation before we work completely through an example computing $SS$, variance, and standard deviation. Like the mean ($\mu$), variance and standard deviation are parameters of a population and will be identified by Greek letters. To identify the standard deviation, we use the Greek letter $\sigma$ (the Greek letter $s$, standing for standard deviation). The capital letter sigma ($\Sigma$) has been used already, so we now use the lowercase sigma, $\sigma$, as the symbol for the population standard deviation. To emphasize the relationship between standard deviation and variance, we use $\sigma^2$ as the symbol for population variance (standard deviation is the square root of the variance). Thus,

$$\text{population standard deviation} = \sigma = \sqrt{\sigma^2} = \sqrt{\frac{SS}{N}} \quad (4.3)$$

$$\text{population variance} = \sigma^2 = \frac{SS}{N} \quad (4.4)$$

Earlier, in Examples 4.3 and 4.4, we computed the sum of squared deviations for a simple population of $N = 4$ scores (1, 0, 6, 1) and obtained $SS = 22$. For this population, the variance is

$$\sigma^2 = \frac{SS}{N} = \frac{22}{4} = 5.50$$

and the standard deviation is $\sigma = \sqrt{5.50} = 2.345$

In frequency distribution graphs, we identify the position of the population mean by drawing a vertical line and labeling it with $\mu$. (Figure 4.4). Because the standard deviation measures distance from the mean, it will be represented by a line or an arrow drawn from the mean outward for a distance equal to the standard deviation (see Figure 4.4). For rough sketches, you can identify the mean with a vertical line in the middle of the distribution. The standard deviation line should extend approximately halfway from the
mean to the most extreme score. \(\textbf{(Note: In Figure 4.4 we show the standard deviation as an arrow pointing to the right. You should realize that we could have drawn the arrow pointing to the left, or we could have drawn two arrows, with one pointing to the right and one pointing to the left. In each case, the goal is to show the standard distance from the mean.)}\)

**FIGURE 4.4**
The graphic representation of a population with a mean of \(\mu = 40\) and a standard deviation of \(\sigma = 4\).

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**LEARNING CHECK**

1. Briefly explain what is measured by the standard deviation and what is measured by the variance.

2. What is the standard deviation for the following set of \(N = 5\) scores: 10, 10, 10, 10, and 10? \(\textbf{(Note: You should be able to answer this question directly from the definition of standard deviation, without doing any calculations.)}\)

3. Find the sum of the squared deviations, \(SS\), for each of the following populations. Note that the definitional formula works well for one population but the computational formula is better for the other.

   Population 1: 3, 1, 5, 1
   Population 2: 6, 4, 2, 0, 9, 3

4. a. Sketch a histogram showing the frequency distribution for the following population of \(N = 6\) scores: 12, 0, 1, 7, 4, 6. Locate the mean in your sketch, and estimate the value of the standard deviation.

   b. Calculate \(SS\), variance, and the standard deviation for these scores. How well does you estimate compare with the actual standard deviation?

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**ANSWERS**

1. Standard deviation measures the standard distance from the mean and variance measures the average squared distance from the mean.

2. Because there is no variability (the scores are all the same), the standard deviation is zero.

3. For population 1, the computational formula is better and produces \(SS = 11\). The definitional formula works well for population 2 and produces \(SS = 50\).

4. a. Your sketch should show a mean of \(\mu = 5\). The scores closest to the mean are \(X = 4\) and \(X = 6\), both of which are only 1 point away. The score farthest from the mean is \(X = 12\), which is 7 points away. The standard deviation should have a value between 1 and 7, probably around 4 points.

   b. For these scores, \(SS = 96\), the variance is \(96/6 = 16\), and the standard deviation is \(\sigma = 4\).
A sample statistic is said to be **biased** if, on the average, it consistently overestimates or underestimates the corresponding population parameter.

The goal of inferential statistics is to use the limited information from samples to draw general conclusions about populations. The basic assumption of this process is that samples should be representative of the populations from which they come. This assumption poses a special problem for variability because samples consistently tend to be less variable than their populations. An example of this general tendency is shown in Figure 4.5. Notice that a few extreme scores in the population tend to make the population variability relatively large. However, these extreme values are unlikely to be obtained when you are selecting a sample, which means that the sample variability is relatively small. The fact that a sample tends to be less variable than its population means that sample variability gives a **biased** estimate of population variability. This bias is in the direction of underestimating the population value rather than being right on the mark. (The concept of a biased statistic is discussed in more detail in Section 4.5.)

Fortunately, the bias in sample variability is consistent and predictable, which means it can be corrected. For example, if the speedometer in your car consistently shows speeds that are 5 mph slower than you are actually going, it does not mean that the speedometer is useless. It simply means that you must make an adjustment to the speedometer reading to get an accurate speed. In the same way, we will make an adjustment in the calculation of sample variance. The purpose of the adjustment is to make the resulting value for sample variance an accurate and unbiased representative of the population variance.

The calculations of variance and standard deviation for a sample follow the same steps that were used to find population variance and standard deviation. Except for minor changes in notation, the first three steps in this process are exactly the same for a sample as they were for a population. That is, calculating the sum of the squared deviations, $SS$, is the same for a sample as it is for a population. The changes in

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**FIGURE 4.5**

The population of adult heights forms a normal distribution. If you select a sample from this population, you are most likely to obtain individuals who are near average in height. As a result, the scores in the sample will be less variable (spread out) than the scores in the population.
notation involve using $M$ for the sample mean instead of $\mu$, and using $n$ (instead of $N$) for the number of scores. Thus, to find the $SS$ for a sample

1. Find the deviation for each score: deviation $= X - M$
2. Square each deviation: squared deviation $= (X - M)^2$
3. Add the squared deviations: $SS = \Sigma(X - M)^2$

These three steps can be summarized in a definitional formula for $SS$:

$$\text{Definitional formula: } SS = \Sigma(X - M)^2 \tag{4.5}$$

The value of $SS$ also can be obtained using a computational formula. Except for minor differences in notation, the computational formula for $SS$ is the same for a sample as it was for a population (see Equation 4.2). Using sample notation, this formula is:

$$\text{Computational formula: } SS = \Sigma X^2 - \frac{(\Sigma X)^2}{n} \tag{4.6}$$

Again, calculating $SS$ for a sample is exactly the same as for a population, except for minor changes in notation. After you compute $SS$, however, it becomes critical to differentiate between samples and populations. To correct for the bias in sample variability, it is necessary to make an adjustment in the formulas for sample variance and standard deviation. With this in mind, sample variance (identified by the symbol $s^2$) is defined as

$$\text{sample variance } = s^2 = \frac{SS}{n - 1} \tag{4.7}$$

Sample standard deviation (identified by the symbol $s$) is simply the square root of the variance.

$$\text{sample standard deviation } = s = \sqrt{s^2} = \sqrt{\frac{SS}{n - 1}} \tag{4.8}$$

Remember, sample variability tends to underestimate population variability unless some correction is made.

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**Example 4.5**

We have selected a sample of $n = 7$ scores from a population. The scores are 1, 6, 4, 3, 8, 7, 6. The frequency distribution histogram for this sample is shown in Figure 4.6.

Before we begin any calculations, you should be able to look at the sample distribution and make a preliminary estimate of the outcome. Remember that standard deviation measures the standard distance from the mean. For this sample the mean is $M = \frac{\Sigma X}{n} = 5$. The scores closest to the mean are $X = 4$ and $X = 6$, both of which are exactly 1 point away. The score farthest from the mean is $X = 1$, which is 4 points away. With the smallest distance from the mean equal to 1 and the largest distance equal to 4, we should obtain a standard distance somewhere around 2.5 (between 1 and 4).
We begin the calculations by finding the value of $SS$ for this sample. Because there are only a few scores and the mean is a whole number ($M = 5$), the definitional formula is easy to use.

However, we will calculate $SS$ using the computational formula. We suggest that you check the calculations by using the definitional formula to verify the answer. The calculations are shown in the following table.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$X^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
</tbody>
</table>

For these scores, $n = 7$, $\Sigma X = 35$, and $\Sigma X^2 = 211$

$$SS = \frac{\Sigma X^2 - (\Sigma X)^2}{n} = \frac{211 - \frac{(35)^2}{7}}{7} = 211 - \frac{175}{7} = \frac{36}{7} = 6$$

$SS$ for this sample is 36. Continuing the calculations,

sample variance $= s^2 = \frac{SS}{n - 1} = \frac{36}{7 - 1} = 6$

Finally, the standard deviation is

$$s = \sqrt{s^2} = \sqrt{6} = 2.45$$

Note that the value we obtained is in excellent agreement with our preliminary prediction (Figure 4.6).

Remember that the formulas for sample variance and standard deviation were constructed so that the sample variability would provide a good estimate of population variability. For this reason, the sample variance is often called estimated population variance, and the sample standard deviation is called estimated population standard deviation. When you have only a sample to work with, the sample variance and standard deviation are the best estimates of the true population values.